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1 The Electric Field

1.1 Coulomb’s Law

$$\vec{F} = k\frac{q_1 q_2}{r^2}\hat{r} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\hat{r}$$

$k = 8.99^9$, permittivity constant $\xi_0 = 8.85 \times 10^{-12}$

1.2 Gauss’s Law

$\vec{A}$ pointing outward from the surface. An inward piercing field is negative flux. An outward piercing field is positive flux. The integral form:

$$\epsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{\text{enc}}$$

If qenc is positive, the net flux is outward; if qenc is negative, the net flux is inward. The deriavative form:

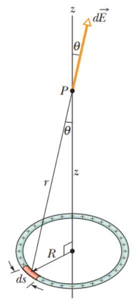
$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon}$$

The electric field due to a charge outside the Gaussian surface contributes zero net flux through the surface, because as many field lines due to that charge enter the surface as leave it.

1.3 The way to find the Electric field

利用库仑定律力的合成叠加原理求场强

1. Ring superposition



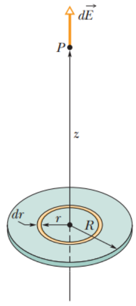
$$\frac{qz}{4\pi\epsilon_0(z^2 + R^2)^{3/2}},$$

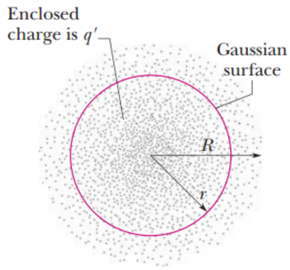
2. Disk superposition

$$\frac{\sigma}{2\epsilon_0}\left[1 - \frac{z}{(z^2 + R^2)^{1/2}}\right]$$

利用高斯定理求场强

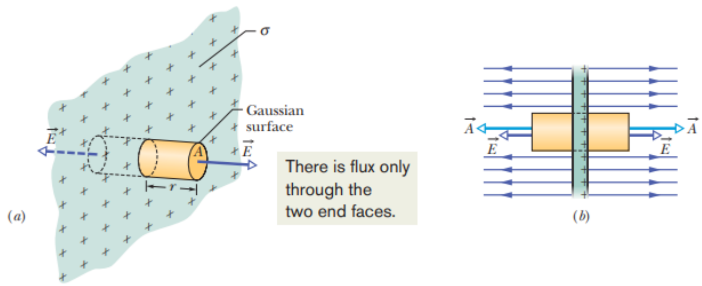
1.Spherical Symmetry: Inside and outside a uniform sphere of charge





$$\begin{cases} \vec{E} = \left(\frac{q}{4\pi\epsilon_0 R^3}\right)\vec{r}, & r \leq R \\ \vec{E} = \left(\frac{q}{4\pi\epsilon_0 r^3}\right)\vec{r}, & r > R \end{cases}$$

2.Planar Symmetry:An infinite sheet with a uniform surface charge density σ.



$$E = \frac{\sigma}{2\epsilon_0}$$

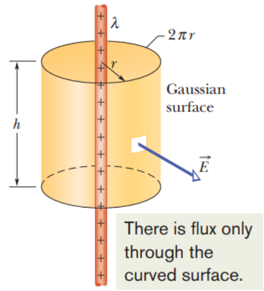
This result holds for any point at a finite distance from the sheet.

3.Cylindrical Symmetry:An infinite line of charge with a uniform line density λ.

$$E = \frac{\lambda h}{\epsilon_0(2\pi r h)} = \frac{\lambda}{2\pi\epsilon_0 r}$$

利用电势求场强

$$E_x = -\frac{\partial V}{\partial x}; \quad E_y = -\frac{\partial V}{\partial y}; \quad E_z = -\frac{\partial V}{\partial z}$$



2 Electric Potential

$$V = \frac{-W}{q} = \frac{U}{q}$$
$$V_f - V_i = -\int_i^F \vec{E} \cdot \mathrm{d}\vec{s} \rightarrow V = -\int_{infinity}^{\vec{r}} \vec{E} \cdot \vec{S} = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$
$$\text{net potential: } V = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i}$$

The total potential energy of systems of charged particles:

$$U_{\text{tot}} = \sum_{i<j} U_{ij} = \frac{1}{4\pi\epsilon_0}\sum_{i<j}\frac{q_i q_j}{r_{ij}} = \frac{1}{8\pi\epsilon_0}\sum_{i\neq j}\frac{q_i q_j}{r_{ij}}$$

Summary

- A charged particle:

$$\frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

- An electric dipole:

$$\frac{1}{4\pi\epsilon_0}\frac{p\cos\theta}{r^2}$$

- A continuous charge distribution (e.g., rod and disk):

$$V = \frac{1}{4\pi\epsilon_0}\int\frac{dq}{r}$$

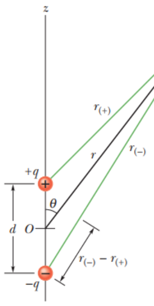
- Electric potential energy of a system of charged particles:

$$U_{\text{tot}} = \sum_{i<j} U_{ij} = \frac{1}{4\pi\epsilon_0}\sum_{i<j}\frac{q_i q_j}{r_{ij}}$$

3 Electric Dipole

Electric dipole moment:

$$\vec{p} = q\vec{d}$$



The electric field at an arbitrary point P along the dipole axis, at distance z from the dipole's center,

$$E = \frac{q}{4\pi\epsilon_0(z-\frac{1}{2}d)^2} - \frac{q}{4\pi\epsilon_0(z+\frac{1}{2}d)^2}$$

$$\Longrightarrow E = \frac{1}{2\pi\epsilon_0}\frac{p}{z^3} \qquad (z \gg d)$$

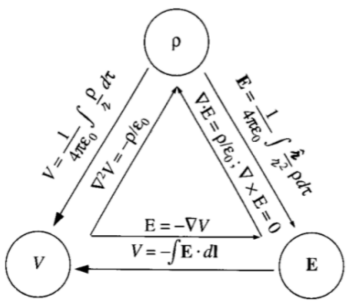
The net torque when a dipole in a uniform electric field:

$$\tau = -Fd\sin\theta = -pEsin\theta = \vec{p}\times\vec{E}$$

Potential due to an Electric Dipole:

$$V = \frac{1}{4\pi\epsilon_0}\frac{p\cos\theta}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{\vec{p}\cdot\vec{r}}{r^3} \qquad (r \gg d)$$

4 The Triangle of Electrostatics



5 The Electrical Properties of Conductors

5.1 A Charged Isolated Conductor

Electrostatic Equilibrium:

$$\boxed{\vec{E}_{inside} = 0, \quad q_{net}^{inside} = 0, \quad V_{inside} = V_{surface}}$$

1. All points of the conductor - whether on the surface or inside - come to the same potential(even if the conductor has an internal cavity and even if that cavity contains a net charge).

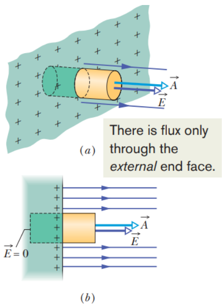
2. All the excess charge remains on the outer surface of the conductor(even if the conductor has an internal cavity).

5.1.1 Electric Field Outside Isolated Conductors

Notice that the surface charge density σ varies, however, over the surface of any nonspherical conductor.

Direction: The electric field $\vec{E}$ at and just outside the conductor's surface must also be perpendicular to that surface.

Cylindrical Gaussian surface:

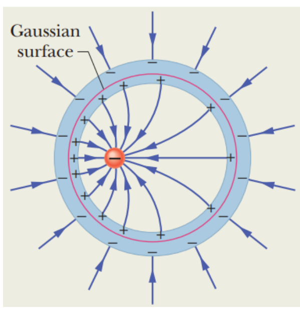


$$\epsilon_0 EA = \sigma A$$
$$\Rightarrow \vec{E} = \frac{\sigma}{\epsilon_0}\hat{n}$$

5.1.2 Parallel Plates

- Single Plate: all the excess charge spreads out on the two faces of the plate with a uniform surface charge density.
- Two Parallel Plates: all the excess charge moves onto the inner faces of the plates.

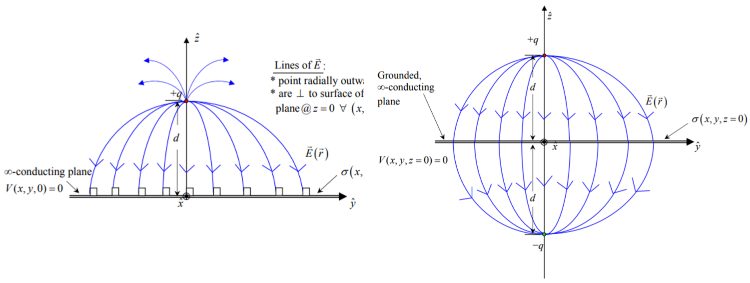
5.2 Charge Inside a Spherical Metal Shell



Charge distribution: a total charge Q must lie on the inner wall of the shell, and a total $-Q$ move to the outer wall and they must spread out uniformly.

5.3 Charge Above a Infinite Grounded Gonducting Plane

The Method of Image



$$V = \frac{q/4\pi\epsilon_0}{\sqrt{x^2+y^2+(z-d)^2}} - \frac{q/4\pi\epsilon_0}{\sqrt{x^2+y^2+(z+d)^2}}.$$

$$\sigma = -\epsilon_0 \left.\frac{\partial V}{\partial z}\right|_{z=0} = -\frac{qd}{2\pi(x^2+y^2+d^2)^{3/2}}.$$

The total charge induced on the z = 0 plane is $-q$.

6 Resistance and Capacitance

6.1 Resistance

Concepts

- Current Density:

$$i = \int \vec{J} d\vec{A}$$

-

• • Drift Velocity : (n is the number of carriers per unit volume)

$$i = nAev_d \qquad \vec{J} = ne\vec{v_d}$$

• Resitivity and conducticivity:

1.

$$\vec{E} = \rho\vec{J} \Rightarrow \rho = \frac{1}{\sigma} = \frac{E}{J}$$

2.

$$\vec{a} = -\frac{e\vec{E}}{m}$$

In the average time τ(mean free time) between collisions, the electro will on average acquire

$$\vec{v}_d = \vec{a}\tau = -\frac{e\vec{E}}{m}\tau \quad \Rightarrow \boxed{\rho = \frac{m}{ne^2\tau}}$$

- Resistance:

$$R = \frac{V}{i} \quad R = \rho\frac{L}{A}$$

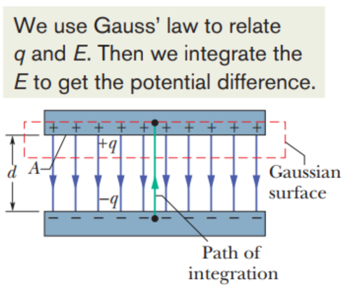
Equation of Continuity

$$\frac{\partial\rho}{\partial t} = \nabla \cdot \vec{J}$$

6.2 Calculating Capacitance

1. Assume a charge q on the plates;
2. calculate the electric field $\vec{E}$ between the plates in terms of this charge, using Gauss' law;
3. knowing $\vec{E}$, calculate the potential difference V between the plates from $V = -\int_-^+ \vec{E} \cdot d\vec{s} = \int_-^+ Eds$ (note the sign);
4. calculate C from $q = CV$.

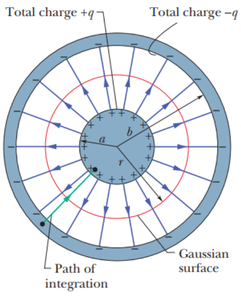
Capacitance of a Parallel-Plate Capacitor:



$$q = \epsilon_0 EA$$
$$C = \frac{\epsilon_0 A}{d}$$

Capacitance of a Cylindrical Capacitor:

$$q = \epsilon_0 EA = \epsilon_0 E(2\pi rL)$$
$$V = -\int_-^+ \vec{E} \cdot d\vec{s} = -\frac{q}{2\pi\epsilon_0 L}\int_b^a \frac{dr}{r} = \frac{q}{2\pi\epsilon_0 L}\ln\left(\frac{b}{a}\right)$$
$$C = \frac{q}{V} = 2\pi\epsilon_0\frac{L}{\ln(b/a)}$$



6.3 Energy Stored in a Capacitor

Energy:

$$U = \frac{q^2}{2C} = \frac{1}{2}CV^2$$

Energy Density:

$$u = \frac{1}{2}\epsilon_0 E^2$$

Gauss' law with a dielectric (with$\vec{D} \equiv \kappa\epsilon_0\vec{E}$)

$$\oint \vec{D} \cdot d\vec{A} = \epsilon_0 \oint \kappa\vec{E} \cdot d\vec{A} = q$$

6.4 DC Circuits

Concepts emf ε, pover P, dielectric constant κ, electric displacement $\vec{D}$

$$\mathcal{E} = \frac{W}{q}$$
$$P = iV$$
$$P = i^2R = \frac{V^2}{R}$$

Kirchhoff ' s loop rule: The algebraic sum of the changes in potential encountered in a complete traversal of any loop of a circuit must be zero.

Charging a Capacitor

$$\frac{dU}{dt} = \frac{q}{C}\frac{dq}{dt} = iE - i^2R.$$

- Noting $i = dq/dt$,we find

$$R\frac{dq}{dt} + \frac{q}{C} = \varepsilon,$$

capacitive time constant$\tau = RC$

$$\Rightarrow q = C\varepsilon(1 - e^{-t/\tau}) \Rightarrow i = \frac{\varepsilon}{R}e^{-t/\tau}$$

Discharging a Capacitor Energy change:

$$\frac{d}{dt}\left(\frac{q^2}{2C}\right) + i^2R = 0$$
$$\frac{dq}{dt} + \frac{q}{RC} = 0$$

这个方程也可以从 Loop Rule 列回路电压变化为 0，电压降等于电压升。

6.5 Dielectrics and Gauss' Law

$$\epsilon_0 \oint \kappa \vec{E} \cdot d\vec{A} = \oint \vec{D} \cdot d\vec{A} = q,$$

electric displacement $\vec{D} \equiv \kappa\epsilon\vec{E}$

7 The Ampere Force

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

利用右手确定方向

7.1 Hlical Movement

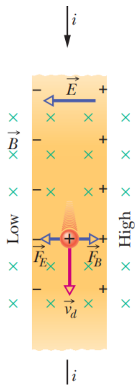
The radius of the helix: $r = \frac{mv_\perp}{|q|B}$.
The parallel component $v_{||}$ determines the pitch p of the helix -that is, the distance between adjacent turns.

7.2 The Hall Effect

$$i = JA = nev_dA$$
$$eE = ev_dB_z$$
$$V = Ed = -v_dBd = -\frac{i}{neA}Bd = -\frac{B}{ne}Jd$$
$$\text{Hall resistivity and coefficient}\rho_{xy} = \frac{E_y}{J_x} = -\frac{B}{ne}, \quad R_H = \frac{E_y}{B_zJ_x} = -\frac{1}{ne}$$

7.3 Current-Carrying Wire

$$\vec{F} = i\vec{L} \times \vec{B}$$
$$\mathrm{d}\vec{F} = i\mathrm{d}\vec{L} \times \vec{B}$$



8 The Magenetic Field

8.1 Biot-Savart law

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{id\vec{s}\times\vec{r}}{r^3},$$

where the constant $0 = 4\times10^{-7}T\cdot m/A$ is called the permeability constant. **Force Between**

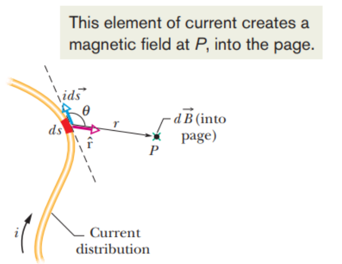


图 1: Biot-Savart Law

Two Parallel Wires $F_{ba} = |i_b\vec{L}\times\vec{B}_a| = \frac{\mu_0 L i_a i_b}{2\pi d}$,

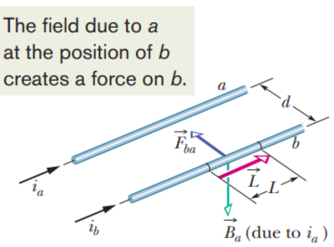
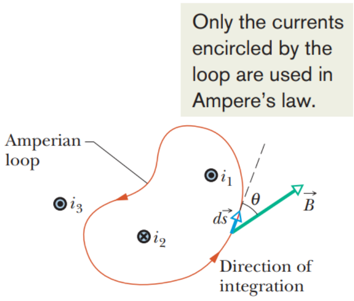


图 2: Force Between Two Parallel Wires



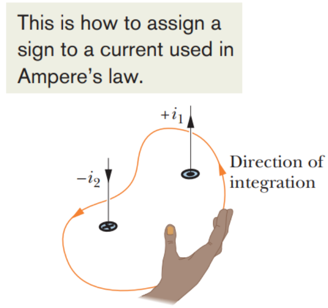


Figure 4: Net current $i_{\text{enc}} = i_1 - i_2$.

8.2 Ampere's Law

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}},$$

where i_{enc} is the net current encircled by the closed loop.

8.3 Examples

8.3.1 A Long Straight Wire

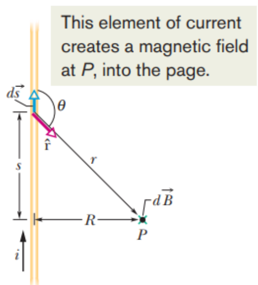


图 3: A Long Straight Wire

$$\begin{aligned} d\vec{B} &= \frac{\mu_0}{4\pi} \frac{i d\vec{s} \times \vec{r}}{r^3} \\ &= \frac{\mu_0}{4\pi} \frac{i d\vec{s} \times \vec{R}}{r^3}. \end{aligned}$$

$$B = \frac{\mu_0}{4\pi} \int_{-\infty}^{\infty} \frac{iRds}{r^3} = \frac{\mu_0 i}{4\pi R} \left[\int_{-\infty}^{\infty} \frac{R^2 ds}{r^3} \right],$$

$$B = \frac{\mu_0 i}{2\pi R},$$

The direction follows a curled-straight right-hand rule(右手螺旋法则).

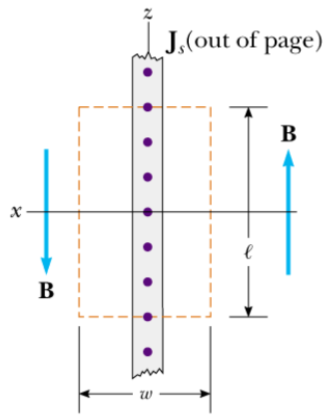


图 4: An infinite Sheet

8.3.2 Outside a Long Straight Wire

$$\oint \vec{B} \cdot \mathrm{d}\vec{S} = B(2\pi r) = \mu_0 i \Rightarrow boxed B = \frac{\mu_0 i}{2\pi r}$$

8.3.3 Inside a Long Straight Wire

Supposed that the current is uniformly distributed over thecross section of the wire,

$$i_{enc} = \frac{r^2}{R^2}$$
$$B = \frac{\mu_0 i_{end}}{2\pi r} = \frac{\mu_0}{2\pi R^2} r$$

8.3.4 A Sheet Of Moving Charge

Consider an infinite flat sheet of current density J_s in the y-direction: Ampere’ s law can be applied to the rectangular path:

$$\oint \vec{B} \cdot d\vec{s} = 2B\ell = \mu_0 (J_s \ell)$$
$$B = \frac{\mu_0 J_s}{2}$$

8.3.5 Magnetic Field of a Solenoid

The field inside the coil is uniform and parallel to the solenoid axis. The magnetic field outside the solenoid is zero.

$$\oint \vec{B} \cdot d\vec{s} = \int_a^b \vec{B} \cdot \mathrm{d}\vec{s} \quad = Bh$$

Let n be the number of turns per unit length of thesolenoid; then the loop encloses nh turns and:

$$i_{enc} = i(nh)$$

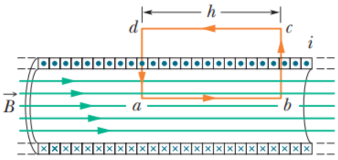


图 5: Amperian Loop of Solenoid

$$\Rightarrow B = \mu_0 in$$

8.3.6 Magnetic Field of a Toroid

A toroid is a solenoid that has been curved until its two ends meet, forming a hollow donut.

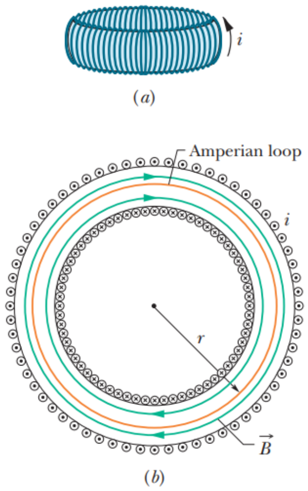


图 6: Amperian Loop of Toroid

$$B = \mu_0 i \frac{N}{2\pi r}$$

B = 0 for points outside an ideal toroid.

8.4 The Properties of B

- The curl of $\vec{B}$: (安培定律的微分形式)

$$\int_S (\nabla \times \vec{B}) \cdot d\vec{A} = \oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}} = \mu_0 \int_S \vec{J} \cdot d\vec{A},$$
$$\nabla \times \vec{B} = \mu_0 \vec{J}(\vec{r})$$

- The Divergence of $\vec{B}$:(高斯定理的微分形式)

$$\oint \vec{B}\cdot d\vec{A} = \int (\nabla\cdot\vec{B})dV = 0.$$

$$\nabla\cdot\vec{B} = 0$$

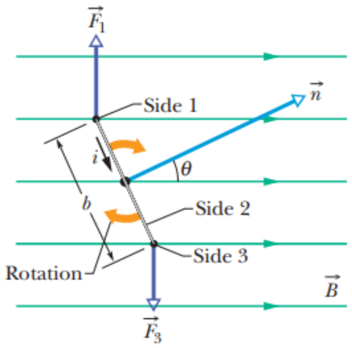
9 Magnetic Properties of Materials

9.1 Magenetic Dipole

The total torque on the coil:

$$\vec{\tau} = Ni\vec{A}\times\vec{B} = \vec{\mu}\times\vec{B},$$

where $\vec{\mu} = Ni\vec{A}$ is known as the magnetic dipole moment of the coil.



$$\tau = -\mu B\sin\theta = -\frac{\partial}{\partial\theta}(-\mu B\cos\theta).$$

The energy of a magenetic dipole:

$$U_B = -\vec{mu}\cdot\vec{B} = -\mu B\cos\theta$$

The field of a Megnetic Dipole The Megneticat field at the center of a single-loop coil with a magnetic dipole moment

$$B = \frac{\mu_0}{2\pi}\frac{\mu}{R^3}$$

The magnetic field at an axial point:

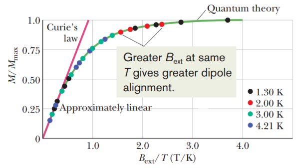
9.2 Magnetic Materials

10 Faraday’ s Law of Induction

$$\mathcal{E} = -N\frac{d\Phi_B}{dt}.$$

Paramagnetism occurs in materials whose atoms have permanent magnetic dipole moments $\vec{\mu}$.
In the absence of an external magnetic field, these atomic dipole moments are randomly oriented, and the net magnetic dipole moment of the material is zero.
In an external magnetic field $\vec{B}_{\text{ext}}$, the magnetic dipole moments tend to line up with the field, which gives the sample a net magnetic dipole moment.

The law is actually an approximation that is valid only when the ratio B_{ext}/T is not too large.



In a sufficiently strong $\vec{B}_{\text{ext}}$, all dipoles in a sample of N atoms and a volume V line up with $\vec{B}$, hence $\vec{M}$ saturates at $M_{\text{max}} = N\mu/V$.

- A paramagnetic solid containing N atoms per unit volume, each atom having a magnetic dipole moment $\vec{\mu}$, with energy U being $-\vec{\mu} \cdot \vec{B}$.
- Suppose the direction of $\vec{\mu}$ can be only parallel or antiparallel to an externally applied magnetic field $\vec{B}$ (this will be the case if $\vec{\mu}$ is due to the spin of a single electron).
- The fraction of atoms whose dipole moment is parallel to $\vec{B}$ is proportional to $e^{-U/k_B T} = e^{\mu B/k_B T}$ and the fraction of atoms whose dipole moment is antiparallel to $\vec{B}$ is proportional to $e^{-\mu B/k_B T}$.
- The magnetization is therefore $e^{\mu B/k_B T} - e^{-\mu B/k_B T} \propto B/T$ for small B/T.

积分形式：

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{A}$$

微分形式：

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t},$$

Notice electric potential has no meaning for electric fields that are produced by induction.

11 Inductors and Inductance

$$L = N\Phi_B/i$$

for a solenoid,

$$L = \mu_0 n^2$$

11.1 RL Circuits

Loop Rule:

$$L\frac{di}{dt} + Ri = \varepsilon$$

inductive time constant: $\tau_L = \frac{L}{R}$

$$i = \frac{\varepsilon}{R}(1 - e^{-t/\tau_L})$$

11.2 Energy and Energy Density

$$U_B = \frac{1}{2}LI^2$$
$$u_b = \frac{B^2}{2\mu_0}$$

11.3 Mutual Induction

Self-inductance L (of a single circuit) and mutual inductance M12 = M21 (of two circuits)

$$\mathcal{E}_{11} = -\frac{d(N_1\Phi_{11})}{dt} = -L_1\frac{di_1}{dt}$$
$$\mathcal{E}_{21} = -\frac{d(N_2\Phi_{21})}{dt} = -M_{21}\frac{di_1}{dt}$$

12 AC Circuits

12.1 LC Oscillations

The total energy U in an oscillating LC circuit is given by

$$U = U_B + U_E = \frac{\mathrm{Li}^2}{2} + \frac{q^2}{2C}.$$

In the absence of resistance, U remains constant with time

$$\frac{dU}{dt} = \frac{d}{dt}\left(\frac{Li^2}{2} + \frac{q^2}{2C}\right) = Li\frac{di}{dt} + \frac{q}{C}\frac{dq}{dt} = 0.$$

With $i = dq/dt$ and $di/dt = d^2q/dt^2$, we find

$$L\frac{d^2q}{dt^2} + \frac{1}{C}q = 0.$$
$$q = Q\cos(\omega_0 t + \phi), \quad \omega = 1/\sqrt{LC}$$

12.2 Damped Oscillations in an RLC Circuit

$$\frac{dU}{dt} = Li\frac{di}{dt} + \frac{q}{C}\frac{dq}{dt} = -i^2R$$
$$\Rightarrow L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = 0$$
$$\Rightarrow q = Qe^{-t/\tau}\cos(\omega t + \phi) \qquad \omega = \sqrt{\omega_0^2 - (1/\tau)^2}$$
$$1/\tau = R/(2L)$$

Forced oscillations in a series RLC circuit at a driving angular frequency ω_d

$$\varepsilon = \varepsilon_m \cos\omega_d t, \quad i = I\cos(\omega_d t + \phi)$$

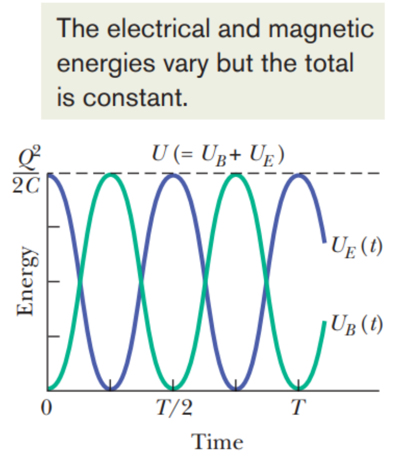


图 7: The Energy Oscillations

12.3 Impendance

Assume the potential difference across a circuit element (resistor, capacitor, and inductor) is

$$v(t) = \Re\left(\tilde{V} e^{i\omega_d t}\right),$$

and the current in the element is

$$i(t) = \Re\left(\tilde{I} e^{i\omega_d t}\right).$$

Define complex impendance as:

$$\tilde{Z} = Z\mathrm{e}^{i\phi} = \frac{\tilde{V}}{\tilde{I}}.$$

注意这里的 i 为虚数,$\Re$ 表示取实部

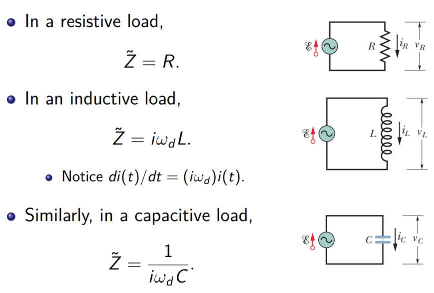


图 8: Three Circuits

13 Maxwell’s Equations and EM Waves

Maxwell’s Equations Gauss’s Law for $\vec{E}$:

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

Faraday's Law:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Gauss's Law for $\vec{B}$

$$\nabla \cdot \vec{B} = 0$$

Ampere-Maxwell's Law:

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

In vacuum, electromagnetic waves satisfy:

$$\nabla^2 \vec{E} = \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2}$$
$$\nabla^2 \vec{B} = \frac{1}{c^2} \frac{\partial^2 \vec{B}}{\partial t^2}.$$

Therefore, in vacuum each Cartesian component of E a $\vec{B}$ satisfies the wave equation

$$\frac{\partial^2 f}{\partial t^2} = c^2 \nabla^2 f,$$

where the speed of all electromagnetic waves is $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \approx 3.00 \times 10^8 \text{m/s}$. Electromagnetic waves are transverse:

$$\hat{k} \cdot \vec{E} = \hat{k} \cdot \vec{B} = 0.$$

$\vec{E}$ is always perpendicular to $\vec{B}$:

$$\vec{B} = \frac{1}{c}(\hat{k} \times \vec{E}).$$